Higher Spin Algebras and Holography in AdS_5 and AdS_7

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Based on joint work with Murat Günaydin
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Motivation

Higher spin/CFT Holography

- Higher spin-CFT duality is the simplest of the AdS/CFT type dualities since the CFT is free.
- Large N critical vector models at fixed points in 3d $\Longleftrightarrow$ Vasiliev HS theories in AdS_4 (Klebanov-Polyakov 02').
- Zoo of parity violating Chern-Simons theories in 3d $\Longleftrightarrow$ Parity violating HS theories in AdS_4 (Giombi, Minwalla ...'09-'14).
- W_N minimal models in 2d $\Longleftrightarrow$ HS theories in AdS_3 (Gaberdiel-Gopakumar '09, $\ldots$).
- $O(N)$ interacting CFTs in 5d $\Longleftrightarrow$ HS theories in AdS_6 (Fei, Giombi, Klebanov '14)
- Maldacena-Zhiboedov ('11) no-go theorem: All CFTs in 3d with exactly conserved higher spin currents must be free theories.

Motivation

Higher spin/CFT Holography

What is missing?

1. HS theories in AdS_5 and AdS_7 and associated dual CFTs.
   - Early conjectures by Sundborg, Sezgin-Sundell ('01) but no concrete tests or recent developments $\Leftarrow$ HS algebras in AdS_5 and AdS_7 are harder and poorly understood.
   - Interacting $O(N)$ vector models exist for $2 < d < 6$ with $d = 4$ excluded so we need to look for possible non-trivial CFT candidates in $d = 4, 6$ that could have interacting HS theories as holographic duals.
2. What about Maldacena-Zhiboedov no-go theorem in higher dimensions?

We believe that understanding the HS algebras better in AdS_5 and AdS_7 is the first step towards answering some of these questions.

Massless fields in AdS and the Boundary

- There exist special representations called *singletons* (*doubletons*) whose Poincaré limit is singular in AdS_d but they describe massless representations of $d-1$ Poincaré group which extend to unitary representations of the $d-1$ conformal group $SO(d-1,2)$.
- Tensor product of two singletons decomposes into an infinite sum of massless representations of AdS_d (but they are massive when restricted to $d-1$ Poincaré group).
- Higher tensor products of singletons/doubletons yield massive AdS_d representations.

Massless fields in AdS and the Boundary

AdS_4	$SO(3,2) \sim Sp(4,\mathbb{R})$	Scalar and spinor singletons (Flato-Fronsdal '78)
AdS_5	$SO(4,2) \sim SU(2,2)$	Infinitely many doubletons (Günaydin-Marcus '84)
AdS_6	$SO(6,2) \sim SO^*(8)$	Infinitely many doubletons (Günaydin-Nieuwenhuizen-Warner '85)

The scalar singleton/doubleton corresponds to the *minimal unitary representation (minrep)*.

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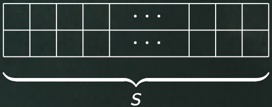
Eastwood-Vasiliev HS algebras

The quotient of universal enveloping algebra (equipped with a Lie bracket) $\mathcal{U}(\mathfrak{so}(d,2))$ *by the annihilator of the minrep (scalar singleton) is the* AdS_d/CFT_{d-1} *HS algebra* $hs(d,2)$. *(Eastwood '02)*

$$hs(d,2) \cong \bigoplus_{s=0}^{\infty}$$



trace-free part

The HS fields in Vasiliev theory are described by two row traceless diagrams and thus it was an indication that $hs(d,2)$ algebra must be $\mathcal{U}(so(d,2))$ quotiented by an ideal. This ideal was identified by Eastwood '02 as the annihilator of the minrep or the Joseph ideal.

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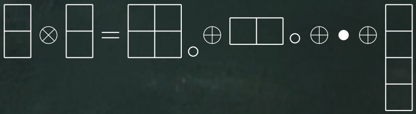
Joseph ideal

The enveloping algebra $\mathcal{U}(\mathfrak{g})$ can be decomposed into standard adjoint action of $\mathfrak{g}$ which by Poincare-Birkhoff-Witt theorem is equivalent to computing symmetric products M_{AB}. In particular, $\bigotimes^2 \mathfrak{so}(d-1,2)$ decomposes as:



The Joseph ideal must remove all the unwanted diagrams from the following except the window diagram. An explicit formula for the generators of Joseph ideal was given by Eastwood, Somberg and Soucek '05

$$\begin{aligned} J_{ABCD} \quad &= \quad M_{AB} \otimes M_{CD} - M_{AB} \circledcirc M_{CD} - \frac{1}{2}\left[M_{AB}\,,\, M_{CD}\right] \\ &\quad + \frac{n-4}{4(n-1)(n-2)} \langle M_{AB}, M_{CD} \rangle \, \mathbf{1} \end{aligned}$$

Joseph ideal in oscillator reps

Joseph ideal is evaluated in singleton representation for $SO(3,2)$ and in doubleton representation for $SO(4,2)$ and $SO(6,2)$.

Algebra	Joseph ideal	HS algebra
$SO(3,2)$	Vanishes identically as an operator	$hs(3,2) = \mathcal{U}(\mathfrak{so}(3,2))$
$SO(4,2)$	Non-vanishing terms proportional to linear Casimir $\mathcal{Z}$	Non-trivial constraints
$SO(6,2)$	Non-vanishing terms proportional to quadratic Casimir of $SU(2)$ subalgebra	Non-trivial constraints

Thus $\mathcal{U}(\mathfrak{so}(4,2))$ and $\mathcal{U}(\mathfrak{so}(6,2))$ in covariant twistorial oscillator representations do not directly yield the $hs(4,2)$ and $hs(6,2)$ and one needs to impose highly non-trivial constraints in order to quotient out the ideal.

Symmetries of scalar singleton/doubleton

- Since the Joseph ideal vanishes on the scalar singleton/doubleton module $\mathcal{D}$ (minrep), HS algebra is just the realization of $\mathcal{U}(\mathfrak{so}(d,2))$ on $\mathcal{D}$.
- In the covariant twistorial oscillator realization of $\mathfrak{so}(4,2)$ and $\mathfrak{so}(6,2)$, one needs to impose constraints on the Fock space to obtain $\mathcal{D}$.
- "Quasiconformal group (QCG) approach" does precisely this and also gives the other modules corresponding to massless conformal fields as *deformations* of the minrep.

Motivation for QCG
Jordan algebras and generalized spacetimes

Motivation: Proposal of Günaydin '75

- Twistor formalism in $4d$: spacetime coordinates x_μ can be represented by 2×2 Hermitian matrices $x = x_\mu \sigma^\mu$
- Hermitian matrices over $\mathbb{C}$ close under symmetric anti-commutator product and form Jordan algebra $J_2^{\mathbb{C}}$

Generalized spacetimes defined by J

Spacetime	Jordan algebra	
Rotation	$Aut(J)$	Automorphism group
Lorentz	$Str_0(J)$	Reduced structure group
Lorentz $\oplus$ Dilatations	$Str(J)$	Structure group
Conformal	$Conf(J)$	Conformal group

Conformal vs Quasiconformal

- Not all simple Lie algebras admit a conformal realization with a natural 3-graded structure with respect to a subalgebra of maximal rank. The groups E_8, F_4 and G_2 do not admit conformal realizations.
- However all simple Lie algebra admit a natural 5-graded decomposition with respect to a subalgebra of maximal rank such that grade ± 2 dimensional subspaces are one dimensional.
- Thus conformal realization of a simple Lie algebra is over the space coordinatized by the underlying Jordan algebra J whereas the quasiconformal realization is over the space $\mathcal{T}$ coordinatized by the elements of Freudenthal triple system $FTS(J)$ over J.
- The quasiconformal groups are the invariance groups of "light-cones" defined by a quartic distance function on $\mathcal{T}$.

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Gradings of SO(4,2) ~ SU(2,2)

Quasiconformal 5-grading

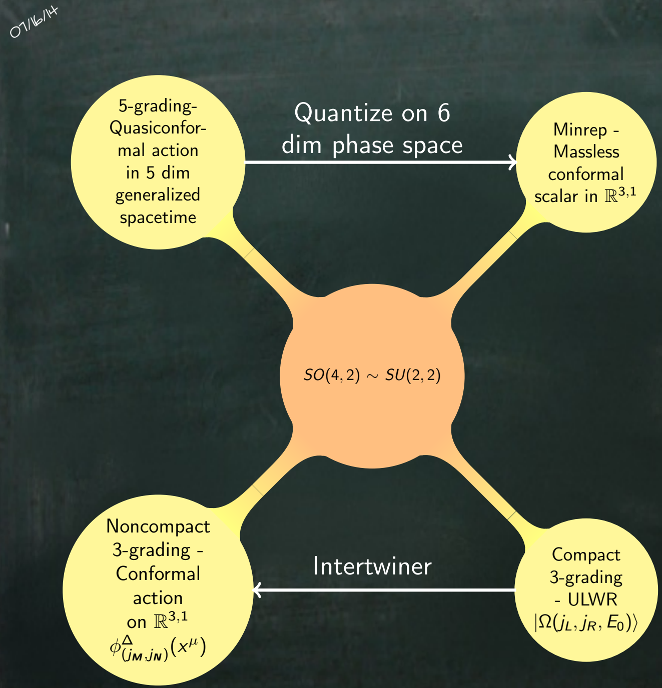
$$so(4,2) = \underset{\mathbf{1}}{\mathfrak{g}^{-2}} \oplus \underset{\mathbf{(2,2)}}{\mathfrak{g}^{-1}} \oplus \underset{\mathbf{SO(2,1)\times SO(2)\times U(1)}}{\mathfrak{g}^{0}} \oplus \underset{\mathbf{(2,2)}}{\mathfrak{g}^{+1}} \oplus \underset{\mathbf{1}}{\mathfrak{g}^{+2}}$$

Conformal (noncompact) 3-grading

$$so(4,2) = \underset{\mathbf{P}_\mu}{\mathfrak{N}^{-1}} \oplus \underset{\mathfrak{sl}(\mathbf{2},\mathbb{C})\times\mathcal{D}}{\mathfrak{N}^{0}} \oplus \underset{\mathbf{K}_\mu}{\mathfrak{N}^{+1}}$$

Compact 3-grading

$$so(4,2) = \underset{\substack{\mathbf{L_{ir}} \\ \textbf{di-annihilation}}}{\mathfrak{C}^{-1}} \oplus \underset{\mathbf{SU(2)_L\times SU(2)_R\times U(1)_E}}{\mathfrak{C}^{0}} \oplus \underset{\substack{\mathbf{L^{ir}} \\ \textbf{di-creation}}}{\mathfrak{C}^{+1}}$$



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5-grading-Quasiconformal action in 5 dim generalized spacetime
Quantize on 6 dim phase space
Minrep - Massless conformal scalar in $\mathbb{R}^{3,1}$
$SO(4,2) \sim SU(2,2)$
Noncompact 3-grading - Conformal action on $\mathbb{R}^{3,1}$ $\phi^{\Delta}_{(j_M,j_N)}(x^\mu)$
Intertwiner
Compact 3-grading - ULWR $|\Omega(j_L, j_R, E_0)\rangle$

Minreps of noncompact groups

- Minreps of noncompact groups and supergroups were obtained in a unified manner by quantization of their quasiconformal realizations by Günaydin and Pavlyk '06.
- The minreps of $SO(4,2)$ and $SO(6,2)$, their supersymmetric extensions and their deformations were first obtained by Fernando and Günaydin ('09-'10) by quasiconformal methods.
- We reformulate these results in terms of *deformed* twistorial oscillators that *transform nonlinearly under the Lorentz group*. We shall see that they are the natural objects to construct $hs(4,2)$, $hs(6,2)$ and their deformations.

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SO(4,2) Realizations
Doubletons: SO(4,2) Covariant Basis

Günaydin, Marcus '85, Günaydin, Minic, Zagermann '98

- Consider bosonic oscillators satisfying: $\left[a_i\,,\,a^j\right]=\delta_i^j,\left[b_i\,,\,b^j\right]=\delta_i^j$ $(i,j=1,2)$.
- We form twistorial Dirac spinor $\Psi=\begin{pmatrix}a_i\\-b^i\end{pmatrix}$ and its conjugate $\overline{\Psi}=\Psi^\dagger\gamma_0=\left(a^i\ b_i\right)$.
- Then the bilinears $M_{AB}=\overline{\Psi}\Sigma_{AB}\Psi$ $(A,B=0,\ldots,5)$ generate the Lie algebra of $SO(4,2)$: $\left[\overline{\Psi}\Sigma_{AB}\Psi\,,\,\overline{\Psi}\Sigma_{CD}\Psi\right]=\overline{\Psi}\left[\Sigma_{AB}\,,\,\Sigma_{CD}\right]\Psi$ where Σ_{AB} are 4×4 matrices satisfying

$$[\Sigma_{AB}\,,\,\Sigma_{CD}]=i(\eta_{BC}\Sigma_{AD}-\eta_{AC}\Sigma_{BD}-\eta_{BD}\Sigma_{AC}+\eta_{AD}\Sigma_{BC})$$

$\eta_{AB}=\mathrm{diag}(-,+,+,+,+,-)$

SO(4,2) Realizations

Doubletons: Compact Basis

$\mathfrak{so}(4,2) \sim \mathfrak{su}(2,2)$ has a 3-grading

$$\mathfrak{su}(2,2) = g^{-1} \oplus g^{0} \oplus g^{+1}$$

where $g^0 = \mathfrak{su}(2)_L \times \mathfrak{su}(2)_R \times \mathfrak{u}(1)_E$ is the maximal compact subalgebra (K). The noncompact generators in g^{-1} and g^{+1} are given as

$$L_{ir} = a_i b_r , \qquad L^{ir} = a^i b^r$$

respectively. They close into compact g^0 generators given by

$$L^i_j = a^i a_j - \frac{1}{2}\delta^i_j (a^k a_k), \qquad R^r_s = b^r b_s - \frac{1}{2}\delta^r_s (b^t b_t)$$

$$E = \frac{1}{2}(a^i a_i + b_r b^r)$$

SO(4,2) Realizations

Lowest energy representations

The positive energy irreducible unitary representations of $SU(2,2)$ are uniquely defined by a "lowest energy representation" $|\Omega\rangle$ transforming irreducibly under K and that is annihilated by L_{ir}

$$L_{ir}\,|\Omega\rangle = 0$$

By acting on $|\Omega\rangle$ repeatedly by L^{ir}, we obtain an infinite set of states $|\Omega\rangle\,, L^{ir}\,|\Omega\rangle\,, L^{ir}L^{js}\,|\Omega\rangle\,, \ldots$ that form the basis of a unitary irrep of $SU(2,2)$.

The states $|\Omega\rangle$ are of the type

$$a^{\dagger}_{i_1} a^{\dagger}_{i_2} \cdots a^{\dagger}_{i_n}\,|0\rangle \quad \Leftrightarrow \quad (j_L, j_R) = \left(\frac{n}{2}, 0\right) \qquad \mathrm{E} = 1 + n/2$$

$$b^{\dagger}_{r_1} b^{\dagger}_{r_2} \cdots b^{\dagger}_{r_m}\,|0\rangle \quad \Leftrightarrow \quad (j_L, j_R) = \left(0, \frac{m}{2}\right) \qquad \mathrm{E} = 1 + m/2$$

$n, m = 0, 1, 2, \ldots$

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SO(4,2) Realizations

Noncompact Basis

As a conformal algebra, $\mathfrak{su}(2,2)$ has a 3-grading w.r.t. dilatations $\mathcal{D}$

$$\mathfrak{su}(2,2) = K_\mu \oplus (SL(2,\mathbb{C}) \times \mathcal{D}) \oplus P_\mu$$

- ULWR in compact basis $\xrightarrow{\text{Intertwiner}}$ Conformal fields in 4d transforming covariantly under $SL(2,\mathbb{C})$ with a definite scale dimension.
- Oscillators $a_i(a^i), b_j(b^j)$ transforming covariantly under K $\xrightarrow{\text{Intertwiner}}$ Covariant oscillators transforming as Weyl spinors (1/2,0) and (0,1/2) of $SL(2,\mathbb{C})$.
- Denoting left and right handed spinors with $(\lambda^\alpha, \eta_\alpha)$ and $(\tilde{\lambda}_{\dot{\beta}}, \eta^{\dot{\alpha}})$ respectively, we get

$$P_{\alpha\dot{\beta}} = -(\sigma^\mu P_\mu)_{\alpha\dot{\beta}} = 2\lambda_\alpha \tilde{\lambda}_{\dot{\beta}} \quad K^{\dot{\alpha}\beta} = -(\bar{\sigma}^\mu K_\mu)^{\dot{\alpha}\beta} = 2\tilde{\eta}^{\dot{\alpha}}\eta^\beta$$

$$\text{where, } [\eta^\alpha, \lambda_\beta] = \delta^\alpha_\beta, \quad \left[\tilde{\eta}^{\dot{\alpha}}, \tilde{\lambda}_{\dot{\beta}}\right] = \delta^{\dot{\alpha}}_{\dot{\beta}}$$

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SO(4,2) Realizations
Coherent states

- The state $|\Omega(j_L, j_R, E_0)\rangle$ is intertwined with a state $T\,|\Omega(j_L, j_R, E_0)\rangle$ that is annihilated by K_μ.
- $\left|\Phi^{\Delta}_{(j_M,j_N)}(0)\right\rangle := T\,|\Omega(j_L, j_R, E_0)\rangle$ transforms irreducibly under the Lorentz group $SL(2,\mathbb{C})$ with quantum numbers $(j_M, j_N) = (j_L, j_R)$ and conformal dimension $\Delta = -E_0$.
- We can create coherent states labeled by coordinates x^μ by acting on $\left|\Phi^{\Delta}_{(j_M,j_N)}(0)\right\rangle$ with P_μ

$$\left|\Phi^{\Delta}_{(j_M,j_N)}(x^\mu)\right\rangle \equiv e^{-ix^\mu P_\mu}\left|\Phi^{\Delta}_{(j_M,j_N)}(0)\right\rangle$$

- The coherent states correspond to states created by action of conformal fields $\Phi^{\Delta}_{(j_M,j_N)}(x^\mu)$ acting on vacuum $|0\rangle$

$$\Phi^{\Delta}_{(j_M,j_N)}(x^\mu)\,|0\rangle \cong \left|\Phi^{\Delta}_{(j_M,j_N)}(x^\mu)\right\rangle$$

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SO(4,2) Realizations

QCG variables

Introduce the variables

$$[x\,,\,p]=i,\qquad \left[d\,,\,d^{\dagger}\right]=1,\qquad \left[g\,,\,g^{\dagger}\right]=1$$

and "singular" oscillators $A_{\mathcal{L}}=\frac{1}{\sqrt{2}}\left(x+ip-\frac{\mathcal{L}}{x}\right),\ A_{\mathcal{L}}^{\dagger}=\frac{1}{\sqrt{2}}\left(x-ip-\frac{\mathcal{L}}{x}\right)$ $(\mathcal{L}=N_d-N_g+1/2)$and nonlinear twistors

$$Z_1=\frac{A_{\mathcal{L}}}{\sqrt{2}}-ig^{\dagger},\quad \widetilde{Z}_{\dot{1}}=(Z_1)^{\dagger},\quad Y^1=-\frac{A_{\mathcal{L}}^{\dagger}}{\sqrt{2}}+ig,\quad \widetilde{Y}^{\dot{1}}=\frac{A_{\mathcal{L}}}{\sqrt{2}}+ig^{\dagger}$$

$$Z_2=-\frac{A_{-\mathcal{L}}^{\dagger}}{\sqrt{2}}-id,\quad \widetilde{Z}_{\dot{2}}=(Z_2)^{\dagger},\quad Y^2=-\frac{A_{-\mathcal{L}}}{\sqrt{2}}-id^{\dagger},\quad \widetilde{Y}^{\dot{2}}=\frac{A_{-\mathcal{L}}^{\dagger}}{\sqrt{2}}-id$$

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SO(4,2) Realizations

QCG generators and deformations

$$P_{\alpha\dot\beta} = -Z_\alpha \tilde Z_{\dot\beta}, \quad K^{\dot\alpha\beta} = -\tilde Y^{\dot\alpha} Y^\beta, \quad \Delta = \frac{i}{2}\left(Z_\alpha Y^\alpha + \tilde Y^{\dot\alpha} \tilde Z_{\dot\alpha}\right)$$

$$\underbrace{M_\alpha{}^\beta = \frac{1}{2}\left(Z_\alpha Y^\beta - \frac{1}{2}\delta_\alpha{}^\beta Z_\gamma Y^\gamma\right) \quad \bar M^{\dot\alpha}{}_{\dot\beta} = -\frac{1}{2}\left(\tilde Y^{\dot\alpha}\tilde Z_{\dot\beta} - \frac{1}{2}\delta^{\dot\alpha}{}_{\dot\beta}\tilde Y^{\dot\gamma}\tilde Z_{\dot\gamma}\right)}_{\text{Massless conformal scalar in 4 dimensions}}$$

Deformations of minrep: $\mathcal{L} \to \mathcal{L}_\zeta = \mathcal{L} + \zeta$, ($\zeta \in \mathbb{R}$ is the helicity)

$$\left[Y^1\,,\,\widetilde Y^1\right] = \frac{1}{2x}(Y^1 - \widetilde Y^1),\ \left[Z_1\,,\,\widetilde Z_1\right] = -\frac{1}{2x}(Z_1 + \widetilde Z_1)$$

The deformed twistors also transform nonlinearly under the Lorentz group.

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Joseph ideal generators

$\zeta/2 =$ Helicity (deformation parameter)

$$\underbrace{P^2 = P^\mu P_\mu = 0}_{\text{4d Masslessness}} \quad , \quad K^2 = K^\mu K_\mu = 0$$

$$\begin{aligned} 4\Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + P^\mu \cdot K_\mu &= 0 \\ P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\Delta) = K^\mu \cdot (M_{\nu\mu} + \eta_{\nu\mu}\Delta) &= 0 \end{aligned}$$

$$\underbrace{W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu M_{\rho\sigma} = \zeta P^\mu,}_{\text{Pauli-Lübanski vector}} \qquad V^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} K_\nu M_{\rho\sigma} = -\zeta K^\mu$$

$$\begin{aligned} \eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 2\eta_{\rho\sigma} &= \frac{\zeta^2}{2}\eta_{\rho\sigma} \\ M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} &= \zeta\epsilon_{\mu\nu\rho\sigma}\Delta \\ \Delta \cdot M_{\mu\nu} + P_{[\mu} \cdot K_{\nu]} &= -\frac{\zeta}{2}\epsilon_{\mu\nu\rho\sigma} M^{\rho\sigma} \end{aligned}$$

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$hs(4,2;\zeta)$ algebras

- Since the Joseph ideal vanishes identically for $\zeta = 0$, $\mathcal{U}(\mathfrak{g})$ for the minrep directly yields $hs(4,2)$.
- However for the deformations of the minrep,  which shows that 4 row diagrams are dual to 2 row diagrams and hence the enveloping algebras of deformations of the minrep yield a continuous one-parameter family of deformations of $hs(4,2)$ which we label as $hs(4,2;\zeta)$.
- This also provides a deformation of the Joseph ideal which has not been discussed in the literature before.

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Realizations of SO(6,2)

Doubletons

Günaydin, Nieuwenhuizen, Warner '85, Fernando, Günaydin, Takemae '01, Günaydin, Takemae '99

- Following exactly as for $SU(2,2)$ we have bosonic oscillators c_i, d_j and their hermitian conjugates c^i, d^j $((i,j = 1,2,3,4))$ transforming covariantly under compact $SU(4) \times U(1)$.
- We can use them to form Dirac spinors $\Psi = \begin{pmatrix} c_i \\ d^i \end{pmatrix}$, $\overline{\Psi} = (c^i,\ -d_i)$.

  Then the bilinears $M_{AB} = \overline{\Psi}\Sigma_{AB}\Psi$ provide a realization of the Lie algebra of $SO(6,2)$
- We can use intertwiner $T = e^{\frac{\pi}{2}M_{06}}$ to obtain twistorial oscillators transforming covariantly under Lorentz group $SL(2,\mathbb{H}) \sim SU^*(4)$ with a definite scale dimension.

Realizations of SO(6,2)

Doubletons: Generators

$SU(2)$ covariant twistorial oscillators:

$$\left[\eta^{\alpha i}\,,\,\lambda_{\beta}^{\;j}\right] = -2\delta^{\alpha}_{\beta}\epsilon^{ij}$$

where $i, j = 1, 2$ and, $\alpha, \beta = 1, 2, 3, 4$ ($\epsilon_{12} = \epsilon^{21} = +1$).
Generators in spinorial basis

$$\left(\Sigma^{\mu}P_{\mu}\right)_{\alpha\beta} = P_{\alpha\beta} = \lambda_{\alpha}^{\;i}\lambda_{\beta}^{\;j}\epsilon_{ij}, \qquad \left(\bar{\Sigma}^{\mu}K_{\mu}\right)^{\alpha\beta} = K^{\alpha\beta} = -\eta^{\alpha i}\eta^{\beta j}\epsilon_{ij}$$

$$M_{\alpha}^{\;\beta} = -\frac{i}{2}\left(\Sigma^{\mu}\bar{\Sigma}^{\nu}\right)_{\alpha}^{\;\beta}M_{\mu\nu}, \quad M_{\alpha}^{\;\beta} = -\frac{1}{2}\left(\lambda_{\alpha}^{\;i}\eta^{\beta j} - \frac{1}{4}\delta^{\beta}_{\alpha}\lambda_{\gamma}^{\;i}\eta^{\gamma j}\right)\epsilon_{ij}$$

$$\Delta = \frac{i}{8}\left(\eta^{\alpha i}\lambda_{\alpha}^{\;j} - \lambda_{\alpha}^{\;i}\eta^{\alpha j}\right)\epsilon_{ij}$$

Just like $SU(2,2)$, Fock space decomposes into infinite irreps corresponding to conformal massless fields in six dimensions.

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Realizations of SO(6,2)

QCG : variables

Introduce the variables $[x\,,\,p]=i$, $[a_m\,,\,a^n]=\delta^n_m$, $[b_m\,,\,b^n]=\delta^n_m$, $(m,n=1,2)$ (Fernando & Günaydin '10). In order to write the generators are bilinears of SU(2) covariant nonlinear twistors, we introduce $Y^{\alpha i}$, $\widetilde{Y}^{\alpha i}$, $Z_\alpha{}^i$ and $\widetilde{Z}_\alpha{}^i$ $(\alpha,\beta=1,2,3,4,\ i,j=1,2)$ that transform nonlinearly under the Lorentz group.

$$Z_1{}^1=b_1-\frac{1}{2}(x-ip)+\frac{1}{x}\left(S_0+\frac{3}{4}\right),\quad Z_1{}^2=a_1-\frac{S_-}{x}$$

$$Y^{11}=a^1+\frac{S_+}{x},\quad Y^{12}=-b^1-\frac{1}{2}(x+ip)+\frac{1}{x}\left(S_0-\frac{3}{4}\right),\ \text{etc.}$$

where $SU(2)_S$ is a subgroup of the little group $SO(4)$ with generators:

$$S_+=a^m b_m,\qquad S_-=b^m a_m,\qquad S_0=\frac{1}{2}(N_a-N_b)$$

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Realizations of SO(6,2)

QCG : Generators

$$P_{\alpha\beta} = Z_\alpha{}^i \widetilde{Z}_\beta{}^j \epsilon_{ij}, \ K^{\alpha\beta} = Y^{\alpha i} \widetilde{Y}^{\beta j} \epsilon_{ij}, \ \Delta = \frac{i}{8}\left(Z_\alpha{}^i \widetilde{Y}^{\alpha j} - Y^{\alpha i} \widetilde{Z}_\alpha{}^j \right) \epsilon_{ij}$$

$$M_\alpha{}^\beta = -\frac{1}{2}\left(Z_\alpha{}^i \widetilde{Y}^{\beta j} - \frac{1}{4}\delta^\alpha_\beta Z_\gamma{}^i \widetilde{Y}^{\gamma j} \right) \epsilon_{ij} = \frac{1}{2}\left(Y^{\beta i} \widetilde{Z}_\alpha{}^j - \frac{1}{4}\delta^\alpha_\beta Y^{\gamma i} \widetilde{Z}_\gamma{}^j \right) \epsilon_{ij}$$

In order to write the generators as bilinears of nonlinear twistors in an $SU(2)$ covariant manner and ensure that they are hermitian, we had to introduce an extra set of twistors $\widetilde{Y}^{\alpha i}$ and $\widetilde{Z}_\alpha{}^i$ because the twistors $Y^{\alpha i}$ and $Z_\alpha{}^i$ are not hermitian by themselves and the set $\widetilde{Y}^{\alpha i}$, $\widetilde{Z}_\alpha{}^i$ contains their hermitian conjugates.

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Realizations of SO(6,2)

QCG : Deformations

The minrep of $SO(6,2)$ corresponds to a $6d$ massless conformal scalar field and just like in $4d$, we can deform the minrep to obtain massless conformal fields with higher spins. We achieve this by introducing P pairs of fermionic oscillators ρ_x and χ_x and adding a *spin* term to $SU(2)_S$ subalgebra of little group $SO(4)$ and extending it to $SU(2)_T$

$$T_+ = S_+ + G_+ = a^m b_m + \rho^x \chi_x$$
$$T_- = S_- + G_- = b^m a_m + \chi^x \rho_x$$
$$T_0 = S_0 + G_0 = \frac{1}{2}(N_a - N_b + N_\rho - N_\chi)$$

The deformed twistors $(Z_t)_\alpha{}^i, (\widetilde{Z}_t)_\alpha{}^i$ and $(Y_t)^{\alpha i}, (\widetilde{Y_t})^{\alpha i}$ are then obtained by simply replacing $SU(2)_S$ generators by $SU(2)_T$ in all the formulas.

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Joseph ideal generators

$$\underbrace{P^2 = P^\mu P_\mu = 0}_{\text{6d Masslessness}} \quad , \quad K^2 = K^\mu K_\mu = 0$$

$$\begin{aligned} 6\Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + 2P^\mu \cdot K_\mu &= 0 \\ P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\Delta) = K^\mu \cdot (M_{\nu\mu} + \eta_{\nu\mu}\Delta) &= 0 \end{aligned}$$

$$A_{\mu\nu\rho} = \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma\delta\tau} M^{[\sigma\delta} \cdot P^{\tau]} = 0, \qquad B_{\mu\nu\rho} = \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma\delta\tau} M^{[\sigma\delta} \cdot K^{\tau]} = 0$$

$$\begin{aligned} \eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 4\eta_{\rho\sigma} &= 0 \\ M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} &= 0 \\ \Delta \cdot M_{\mu\nu} + P_{[\mu} \cdot K_{\nu]} &= 0 \end{aligned}$$

Joseph ideal generators

$$\underbrace{P^2 = P^\mu P_\mu = 0}_{\text{6d Masslessness}} \quad , \quad K^2 = K^\mu K_\mu = 0$$

$$\begin{aligned} 6\Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + 2P^\mu \cdot K_\mu &= 0 \\ P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\Delta) = K^\mu \cdot (M_{\nu\mu} + \eta_{\nu\mu}\Delta) &= 0 \end{aligned}$$

$$A_{\mu\nu\rho} = \widetilde{A}_{\mu\nu\rho}, \qquad B_{\mu\nu\rho} = -\widetilde{B}_{\mu\nu\rho}$$

$$M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} = \epsilon_{\mu\nu\rho\sigma}{}^{\delta\tau}(P_{[\delta} \cdot K_{\tau]} + M_{\delta\tau} \cdot \Delta)$$

$$\eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 4\eta_{\rho\sigma} = 2\mathcal{G}^2 \eta_{\rho\sigma}$$

$\mathcal{G}^2$ is the quadratic Casimir of "deformation" $SU(2)_G$ generated by

$$G_+ = \rho^x \chi_x \qquad G_- = \chi^x \rho_x \qquad G_0 = \frac{1}{2}(N_\rho - N_\chi)$$

hs(6,2 ;t) algebras

- The Joseph ideal vanishes identically for minrep $\implies$ $\mathcal{U}(\mathfrak{g})$ for the minrep directly yields $hs(6,2)$.

- However for the deformations of the minrep,  doesn't vanish

  but satisfies an 8-dimensional self-duality condition c.f. 3-form gauge fields with self-dual field strengths.

- This is analogous to the 3-form field that descends from 11d SUGRA on $AdS_7 \times S^4$. In 6d, they correspond to conformal 2-form fields with a self dual field strength which is simply the tensor field of $(2,0)$ conformal supermultiplet.

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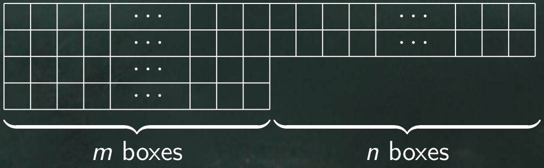
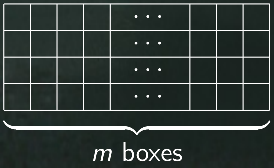
hs(6,2 ;t) algebras

Thus $hs(6, 2; t)$ algebra generators include Young tableaux of the form



This suggests that higher spin theories based on discrete deformations of the minrep describe higher spin theory of Vasiliev type in AdS_7 coupled to tensor fields that satisfy self-duality conditions and their higher extensions corresponding to the Young tableaux



Supersymmetric extensions

- The QCG approach to minreps and deformations have straightforward supersymmetric extensions. One introduces a set of fermionic oscillators to realize the R-symmetry algebra and the odd generators are realized as bilinears of nonlinear (deformed) twistors and fermionic oscillators.
- HS algebras $hsu(2,2|N)$ and $hosp(8^*|2N)$ are then given by $\mathcal{U}(\mathfrak{su}(2,2|N))$ and $\mathcal{U}(\mathfrak{osp}(8^*|2N))$ of their minreps obtained by QCG approach. They admit one-parameter deformations $hsu(2,2|N,\zeta),(\zeta \in \mathbb{R})$ and $hosp(8^*|2N,t)(t = (2k+1)/2, k \in \mathbb{N})$.
- Note that this algebra contains as subalgebras the HS algebras of different representations of $SU(2,2)$ and $SO^*(8)$ that form the supermultiplet.

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AdS_5/CFT_4 higher spin holography

- In the large N limit of CFT_4, correlation functions factorize as products of 2-point functions which corresponds to tensor product of doubletons $\Leftrightarrow$ Massless fields in the bulk AdS_5. (Mikhailov '02).
- Similar arguments suggest existence of a 1-parameter family of supersymmetric higher spin theories of massless fields in AdS_5.
- Moreover, the minrep and deformations are nonlinear realizations $\Rightarrow$ Bulk theory must be interacting (at least for $\zeta \neq \mathbb{Z}$).
- Evidence in $d = 1$: Symmetry algebras of interacting (nonlinear) superconformal quantum mechanical models of (Fedoruk, Ivanov & Lechtenfeld '09) furnish a 1-parameter family of deformations of the minrep of $\mathcal{N} = 4$ superconformal algebra $D(2,1;\lambda)$(KG, Günaydin '12, Günaydin '07)

AdS_7/CFT_6 higher spin holography

- Our results suggest the existence of a family of (supersymmetric) interacting higher spin theories in AdS_7 that are dual to free (supersymmetric) CFT's (interacting?) in six dimensions.
- Of particular interest are the higher spin superalgebras based on $OSp(8^*|4)$ and $OSp(8^*|8)$ whose minimal unitary supermultiplets reduce to $N = 4$ Yang-Mills supermultiplet and $N = 8$ supergravity multiplet under dimensional reduction to four dimensions.
- The minimal unitary supermultiplet of $OSp(8^*|4)$ is the $6d$ (2,0) conformal tensor multiplet whose interacting theory is believed to be dual to M-theory over $AdS_7 \times S^4$. Our results suggest that there exists a limit of this interacting theory that is dual to a higher spin theory in AdS_7.

Conclusions and open problems

- The minreps and their deformations obtained from quasiconformal methods provide a natural framework for defining HS algebras in AdS_5 and AdS_7 and providing their unitary realizations.
- Our results suggest existence of infinitely many HS interacting theories in AdS_5 and AdS_7.
- The next step is to understand the physical content of HS gauge theories based on these algebras and the characteristics of dual CFTs.
- An important issue is the physical meaning of the deformed twistors and developing a star-product calculus for them to construct Vasiliev type theories in AdS_5 and AdS_7.
- Another direction would be to reformulate the spin chain models associated with $\mathcal{N} = 4$ SYM in terms of deformed twistors and study the integrability of corresponding spin chains non-perturbatively.

Thank you for your attention!