On Kostant's theorem for the Lie superalgebra $Q(n)$

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1. Introduction

- A *finite W-algebra* is a certain associative algebra attached to a pair $(\mathfrak{g}, e)$ where $\mathfrak{g}$ is a complex semi-simple Lie algebra and $e \in \mathfrak{g}$ is a nilpotent element.

- It is a quantization of the Poisson algebra of functions on the Slodowy slice at e to the orbit $Ad(G)e$, where $\mathfrak{g} = Lie(G)$.

- Due to recent results of I. Losev, A. Premet and others, finite W-algebras play a very important role in description of primitive ideals.

- Finite W-algebras for semi-simple Lie algebras were introduced by A. Premet.

- Finite W-algebras for Lie algebras and superalgebras have been extensively studied by mathematicians and physicists: L. Fehér, C. Briot, E. Ragoucy, A. Premet, I. Losev, V. Ginzburg, W. L. Gan, J. Brundan, J. Brown, S. Goodwin, W. Wang, L. Zhao, Y. Zeng, B. Shu.

2. Finite W-algebras for Lie superalgebras

Let $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ be a Lie superalgebra with reductive even part $\mathfrak{g}_{\bar{0}}$,

$\chi \in \mathfrak{g}_{\bar{0}}^* \subset \mathfrak{g}^*$ be an even element in the coadjoint representation,

$G_{\bar{0}}$ be the algebraic reductive group of $\mathfrak{g}_{\bar{0}}$.

Definition 2.1. *χ is* **nilpotent** *if the closure of the $G_{\bar{0}}$-orbit in $\mathfrak{g}_{\bar{0}}^*$ contains zero.*

Let $\mathfrak{g}^\chi$ be the **annihilator** of χ in $\mathfrak{g}$:

$$\mathfrak{g}^\chi = \{x \in \mathfrak{g} \mid \chi([x, \mathfrak{g}]) = 0\}$$

Definition 2.2. A good $\mathbb{Z}$-grading *for χ is a $\mathbb{Z}$-grading $\mathfrak{g} = \bigoplus_{j \in \mathbb{Z}} \mathfrak{g}_j$ satisfying the following two conditions:*

(1) $\chi(\mathfrak{g}_j) = 0$ *if* $j \neq -2$,

(2) $\mathfrak{g}^\chi$ *belongs to* $\bigoplus_{j \geq 0} \mathfrak{g}_j$.

- $\chi([\cdot,\cdot]) : \mathfrak{g}_{-1} \times \mathfrak{g}_{-1} \to \mathbb{C}$ is a non-degenerate (super)skew-symmetric even bilinear form on $\mathfrak{g}_{-1}$.

- Let $\mathfrak{l}$ be a maximal isotropic (Lagrangian) subspace with respect to this form.

Let $\mathfrak{m} = (\bigoplus_{j \le -2} \mathfrak{g}_j) \oplus \mathfrak{l}$. The restriction of χ to $\mathfrak{m}$

$$\chi : \mathfrak{m} \longrightarrow \mathbb{C}$$

defines a one-dimensional representation $C_\chi =< v >$ of $\mathfrak{m}$.

Definition 2.3. *The induced $\mathfrak{g}$-module*

$$Q_\chi := U(\mathfrak{g}) \otimes_{U(\mathfrak{m})} C_\chi \cong U(\mathfrak{g})/I_\chi,$$

where I_χ is the left ideal of $U(\mathfrak{g})$ generated by $a - \chi(a)$ for all $a \in \mathfrak{m}$,

is called the **generalized Whittaker module***.*

Definition 2.4. The finite W-algebra *associated to the nilpotent element χ is*

$$W_\chi := End_{U(\mathfrak{g})}(Q_\chi)^{op}$$

- W_χ can be identified with the subspace

$$Q_\chi^{\mathfrak{m}} = \{u \in Q_\chi \mid au = \chi(a)u \text{ for all } a \in \mathfrak{m}\}$$

- Let $\pi : U(\mathfrak{g}) \to U(\mathfrak{g})/I_\chi$ be the natural projection. Then

$$W_\chi = \{\pi(y) \in U(\mathfrak{g})/I_\chi \mid \mathrm{ad}(a)y \in I_\chi \text{ for all } a \in \mathfrak{m}\}$$

The algebra structure on W_χ is given by

$$\pi(y_1)\pi(y_2) = \pi(y_1 y_2)$$

for $y_i \in U(\mathfrak{g})$ such that $\mathrm{ad}(a)y_i \in I_\chi$ for all $a \in \mathfrak{m}$ and $i = 1, 2$.

• As in the Lie algebra case, the superalgebras W_χ are all isomorphic for different choices of good $\mathbb{Z}$-gradings and maximal isotropic subspaces $\mathfrak{l}$.

• If $\mathfrak{g}$ admits an even non-degenerate invariant supersymmetric bilinear form, then $\mathfrak{g} \simeq \mathfrak{g}^*$ and $\chi(x) = (e|x)$ for some nilpotent $e \in \mathfrak{g}_{\bar{0}}$.

By the Jacobson–Morozov theorem e can be included in $\mathfrak{sl}(2) =< e, h, f >$. The linear operator $\mathrm{ad}h$ defines a Dynkin $\mathbb{Z}$-grading $\mathfrak{g} = \bigoplus_{j\in\mathbb{Z}} \mathfrak{g}_j$, where

$$\mathfrak{g}_j = \{x \in \mathfrak{g} \mid \mathrm{ad}h(x) = jx\}.$$

The Dynkin $\mathbb{Z}$-grading is good for χ.

Example 2.5. *Let* $\chi = 0$. *Then* $\mathfrak{m} = 0$,

$$Q_\chi = U(\mathfrak{g}), \quad W_\chi = U(\mathfrak{g}).$$

Definition 2.6. *A nilpotent* $\chi \in \mathfrak{g}_{\bar{0}}^*$ *is called* **regular** *if the* $G_{\bar{0}}$*-orbit of* χ *has maximal dimension, i.e. the dimension of* $\mathfrak{g}_{\bar{0}}^\chi$ *is minimal.*

Theorem 2.7. *B. Kostant (1978)*

For a regular nilpotent χ *and a reductive Lie algebra* $\mathfrak{g}$ *the algebra* W_χ *is isomorphic to the center of* $U(\mathfrak{g})$.

- Theorem of Kostant does not hold for Lie superalgebras since W_χ must have a non-trivial odd part, and the center of $U(\mathfrak{g})$ is even.

Definition 2.8. *Kazhdan filtration on* W_χ.

Define the $\mathbb{Z}$*-grading on* $T(\mathfrak{g})$ *induced by the shift by* 2 *of the fixed good* $\mathbb{Z}$*-grading.*

For $X \in \mathfrak{g}_j$ *set*

$$\deg X = j + 2.$$

This induces a filtration on $U(\mathfrak{g})$*, and therefore on* $U(\mathfrak{g})/I_\chi$ *and on* $W_\chi \subset U(\mathfrak{g})/I_\chi$.

Theorem 2.9. *A. Premet.*

Let $\mathfrak{g}$ *be a semi-simple Lie algebra. Then the associated graded algebra* $Gr(W_\chi)$ *is isomorphic to* $S(\mathfrak{g}^\chi)$.

Conjecture 2.10. *(P-S)*

Assume that $\mathfrak{g}$ *is a Lie superalgebra with reductive even part* $\mathfrak{g}_{\bar{0}}$.

If $dim(\mathfrak{g}_{-1})_{\bar{1}}$ *is even, then* $Gr_K W_\chi \simeq S(\mathfrak{g}^\chi)$

If $dim(\mathfrak{g}_{-1})_{\bar{1}}$ *is odd, then* $Gr_K W_\chi \simeq S(\mathfrak{g}^\chi) \otimes \mathbb{C}[\xi]$,

where $\mathbb{C}[\xi]$ *is the exterior algebra generated by one element* ξ.

- We proved this Conjecture if $\mathfrak{g} = \mathbf{Q}(\mathbf{n})$ or $\mathbf{D}(\mathbf{2}, \mathbf{1}; \alpha)$ and χ is regular.

- Y. Zheng and B. Shu have recently proved this Conjecture for basic classical Lie superalgebras, except for $\mathbf{D}(\mathbf{2}, \mathbf{1}; \alpha)$ (May 2014).

• C. Briot and E. Ragoucy observed that finite W-algebras associated with certain nilpotent orbits in $\mathfrak{gl}(pm|pn)$ can be realized as truncations of the super-Yangian of $\mathfrak{gl}(m|n)$ (2003).

• The finite W-algebras for $\mathfrak{g} = \mathfrak{gl}(m|n)$ associated to **regular** nilpotent elements were described as certain truncations of a shifted version of the super-Yangian $Y(\mathfrak{gl}(1|1))$ by J. Brown, J. Brundan and S. Goodwin (2012).

• We described finite W-algebras associated to **regular** nilpotent elements in terms of generators and relations:

for classical Lie superalgebras of type I and defect one (2011),

for $\mathbf{D(2,1;}\alpha\mathbf{)}$ (2013),

and now for the queer Lie superalgebras $\mathbf{Q(n)}$ (2014).

We also obtained partial results for $\mathfrak{osp}(1|2n)$ and the exceptional Lie superalgebra $F(4)$.

3. The case of $\mathfrak{g} = \mathbf{Q(n)}$

$$Q(n) = \{\left(\begin{array}{c|c} A & B \\ \hline B & A \end{array}\right) \mid A, B \text{ are } n \times n \text{ matrices}\}$$

$e_{i,j}$ and $f_{i,j}$ are standard bases in A and B respectively:

$$e_{i,j} = \left(\begin{array}{c|c} E_{ij} & 0 \\ \hline 0 & E_{ij} \end{array}\right), \quad f_{i,j} = \left(\begin{array}{c|c} 0 & E_{ij} \\ \hline E_{ij} & 0 \end{array}\right)$$

$z = \sum_{i=1}^{n} e_{i,i}$ is a central element

$Q(n)$ admits an **odd** nondegenerate $\mathfrak{g}$-invariant super symmetric bilinear form

$$(x|y) := otr(xy) \text{ for } x, y \in \mathfrak{g},$$

where $otr\left(\begin{array}{c|c} A & B \\ \hline B & A \end{array}\right) = trB.$

Let $\mathfrak{sl}(2) =< e, h, f >$, where

$$e = \sum_{i=1}^{n-1} e_{i,i+1}, \quad h = \mathrm{diag}(n-1, n-3, \ldots, 3-n, 1-n), \quad f = \sum_{i=1}^{n-1} i(n-i) e_{i+1,i}.$$

e is a **regular** nilpotent element.

h defines an **even** Dynkin $\mathbb{Z}$-grading of $\mathfrak{g}$ whose degrees on the elementary matrices are

$$\left(\begin{array}{cccc|cccc}
0 & 2 & \cdots & 2n-2 & 0 & 2 & \cdots & 2n-2 \\
-2 & 0 & \cdots & 2n-4 & -2 & 0 & \cdots & 2n-4 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
2-2n & \cdots & \cdots & 0 & 2-2n & \cdots & \cdots & 0 \\
\hline
0 & 2 & \cdots & 2n-2 & 0 & 2 & \cdots & 2n-2 \\
-2 & 0 & \cdots & 2n-4 & -2 & 0 & \cdots & 2n-4 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
2-2n & \cdots & \cdots & 0 & 2-2n & \cdots & \cdots & 0
\end{array}\right)$$

Replace $e = \sum_{i=1}^{n-1} e_{i,i+1}$ by $E = \sum_{i=1}^{n-1} f_{i,i+1} \Rightarrow E$ is odd:

$$e = \left(\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \hline 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{array}\right), E = \left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \hline 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{array}\right)$$

Let $\chi \in \mathfrak{g}_{\bar{0}}^* \subset \mathfrak{g}^*$ be defined by $\chi(x) = (x|E)$.

$$\mathfrak{g}^\chi = \mathfrak{g}^E = \{z, e, e^2, \ldots, e^{n-1} \mid H_0, H_1, \ldots, H_{n-1}\}, \quad \dim(\mathfrak{g}^E) = (n|n)$$

where $H_0 = \sum_{i=1}^{n}(-1)^{i+1} f_{i,i}$, $H_1 = \sum_{i=1}^{n-1}(-1)^{i} f_{i,i+1}$, $\ldots$ $H_{n-1} = (-1)^{n+1} f_{1,n}$

$$\mathfrak{m} = \bigoplus_{j=2}^{n} \mathfrak{g}_{2-2j}$$

- The left ideal I_χ and W_χ are defined as usual. Moreover,

$$\mathfrak{p} := \bigoplus_{j=0}^{n-1} \mathfrak{g}_{2j}$$

is a parabolic subalgebra of $\mathfrak{g}$.

- Since the good $\mathbb{Z}$-grading is even, then W_χ can be regarded as a *subalgebra* of $U(\mathfrak{p})$.

• A. Sergeev defined by induction the elements $e_{i,j}^{(m)}$ and $f_{i,j}^{(m)}$ belonging to $U(Q(n))$:

$$e_{i,j}^{(m)} = \sum_{k=1}^{n} e_{i,k} e_{k,j}^{(m-1)} + (-1)^{m+1} \sum f_{i,k} f_{k,j}^{(m-1)},$$

$$f_{i,j}^{(m)} = \sum_{k=1}^{n} e_{i,k} f_{k,j}^{(m-1)} + (-1)^{m+1} \sum f_{i,k} e_{k,j}^{(m-1)}$$

Theorem 3.1. *(P–S)*

(1) W_χ *has* n *even generators:* $\pi(e_{n,1}^{(n+k-1)})$, $k = 1, \ldots, n$ *and*

n *odd generators:* $\pi(f_{n,1}^{(n+k-1)})$, $k = 1, \ldots, n$.

(3) $Gr_K W_\chi \simeq S(\mathfrak{g}^\chi)$.

(2) We can construct a set of generators of W_χ *such that even generators commute, and the commutators of odd generators are in the center of* $U(Q(n))$.

Conjecture 3.2. *(P–S) Let* $\mathfrak{g}$ *be a basic classical Lie superalgebra and let* χ *be regular nilpotent. Then it is possible to find a set of generators of* W_χ *such that even generators commute, and the commutators of odd generators are in the center of* $U(\mathfrak{g})$.

4. Super-Yangian of $\mathbf{Q(n)}$

• Super-Yangian $Y(Q(n))$ was studied by M. Nazarov and A. Sergeev.

• $Y(Q(n))$ is the associative unital superalgebra over $\mathbb{C}$ with the countable set of generators

$$T_{i,j}^{(m)} \text{ where } m = 1, 2, \dots \text{ and } i, j = \pm 1, \pm 2, \dots, \pm n$$

The $\mathbb{Z}_2$-grading of the algebra $Y(Q(n))$:

$$p(T_{i,j}^{(m)}) = p(i) + p(j), \text{ where } p(i) = 0 \text{ if } i > 0 \text{ and } p(i) = 1 \text{ if } i < 0$$

Theorem 4.1. *(P–S) There exists a surjective homomorphism:*

$$\varphi : Y(Q(1)) \longrightarrow W_\chi$$

defined as follows:

$$\varphi(T_{1,1}^{(k)}) = (-1)^k \pi(e_{n,1}^{(n+k-1)}), \quad \varphi(T_{-1,1}^{(k)}) = (-1)^k \pi(f_{n,1}^{(n+k-1)}), \text{ for } k = 1, 2, \dots$$

5. The Harish-Chandra homomorphism

- $\mathfrak{g} = Q(n)$, $\chi \in \mathfrak{g}_{\bar{0}}^*$ is nilpotent, $\mathfrak{p} = \bigoplus_{i \geq 0} \mathfrak{g}_i$ is a parabolic subalgebra.
- Consider the grading on $U(\mathfrak{p})$ induced by the even $\mathbb{Z}$-grading of $\mathfrak{g}$. Then

$$U(\mathfrak{p})^+ := \bigoplus_{i>0} U(\mathfrak{p})_i$$

is a two sided ideal in $U(\mathfrak{p})$ and $U(\mathfrak{p})/U(\mathfrak{p})^+ \cong U(\mathfrak{g}_0)$.

- Let $\vartheta : U(\mathfrak{p}) \longrightarrow U(\mathfrak{g}_0)$ be the natural projection.

Since the $\mathbb{Z}$-grading is even, then W_χ is a subalgebra of $U(\mathfrak{p})$.

Theorem 5.1. *(P–S) The restriction of ϑ on W_χ is injective.*

6. The case of a regular nilpotent χ

• If χ is **regular**, then $\mathfrak{g}_0 = \mathfrak{h}$ is the Cartan subalgebra of $\mathfrak{g}$.

$$\mathfrak{h} =< x_i, \xi_i \mid i = 1, \ldots, n >, \text{ where } x_i = e_{i,i},\ \xi_i = f_{i,i}$$

x_i lie in the center of $U(\mathfrak{h})$ and $[\xi_i, \xi_i] = 2x_i$.

• $U(\mathfrak{h}_{\bar{0}}) = \mathbb{C}[x_1, \ldots, x_n]$ coincides with the center of $U(\mathfrak{h})$.

Theorem 6.1. *(P–S) The center of W_χ coincides with the center of $U(Q(n))$.*

Idea of Proof. The center of $\vartheta(W_\chi)$ is $\vartheta(W_\chi) \cap U(\mathfrak{h}_{\bar{0}})$.

Theorem 6.2. *(P–S) Let M be a simple W_χ-module. Then*

$$\dim M \leq 2^{k+1}, \text{ where } k = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ \frac{n-1}{2} & \text{if } n \text{ is odd.} \end{cases}$$

The proof is based on the *Amitsur–Levitzki theorem.*

Theorem 6.3. *(A–L) If $A_1, \dots, A_{2n}$ are $n \times n$ matrices, then*

$$\sum_{\sigma \in S_{2n}} sgn(\sigma) A_{\sigma(1)} \dots A_{\sigma(2n)} = 0.$$

Idea of Proof.

(1) $U(\mathfrak{h})$ satisfies Amitsur–Levitzki identity, i.e. for any $u_1, \dots, u_{2^{k+1}} \in U(\mathfrak{h})$

$$\sum_{\sigma \in S_{2^{k+1}}} sgn(\sigma) u_{\sigma(1)} \dots u_{\sigma(2^{k+1})} = 0. \qquad (*)$$

(2) W_χ satisfies Amitsur–Levitzki identity, since $W_\chi \cong \vartheta(W_\chi) \subset U(\mathfrak{h})$.

(3) Consider M as a module over the associative algebra W_χ, forgetting the $\mathbb{Z}_2$-grading. Then either M is simple or M is a direct sum of two non-homogeneous simple submodules $M_1 \oplus M_2$.

(a) In the former case $\dim M \leq 2^k$.

Assume $\dim M > 2^k$. Let V be a subspace of dimension $2^k + 1$. By density theorem for any $X_1, \dots, X_{2^{k+1}} \in \mathrm{End}_{\mathbb{C}}(V)$ one can find $u_1, \dots, u_{2^{k+1}}$ in W_χ such that $(u_i)_{|V} = X_i$ for all $i = 1, \dots, 2^{k+1}$. Since $\mathrm{End}_{\mathbb{C}}(V)$ does not satisfy (*) we obtain a contradiction.

(b) In the latter case, we can prove in the same way that $\dim M_1 \leq 2^k$ and $\dim M_2 \leq 2^k$. Therefore $\dim M \leq 2^{k+1}$.

Theorem 6.4. *(P–S)*
For a basic classical Lie superalgebra or $Q(n)$ and a regular nilpotent χ, all irreducible representations of W_χ are finite-dimensional:

$$\dim M \leq 2^{k+1}$$

Let d be the **defect** of $\mathfrak{g}$.
In other cases we set $k = d$ or $k = d + 1$.

- $k = d$, if $\mathfrak{g}$ is of type I: $\mathfrak{g} = \mathfrak{sl}(m|n), \mathfrak{osp}(2|2n)$,

  or $\mathfrak{g}$ is of type II and $\dim(\mathfrak{g}_{\bar{1}}^{\chi})$ is even: $\mathfrak{g} = \mathfrak{osp}(2m+1|2n)$ for $m \geq n$,
  $\mathfrak{osp}(2m|2n)$ for $m \leq n$, G_3.

- $k = d + 1$, if $\mathfrak{g}$ is of type II and $\dim(\mathfrak{g}_{\bar{1}}^{\chi})$ is odd:

  $\mathfrak{g} = \mathfrak{osp}(2m+1|2n)$ for $m < n$, $\mathfrak{osp}(2m|2n)$ for $m > n$, $D(2,1;\alpha)$, F_4.

Idea of Proof.

1) If the good $\mathbb{Z}$-grading of $\mathfrak{g}$ with respect to χ is **even**,
then there is an injective homomorphism

$$\vartheta : W_\chi \longrightarrow U(\mathfrak{g}_0).$$

2) If the good $\mathbb{Z}$-grading is **odd**,
then there is an injective homomorphism

$$\vartheta : W_\chi \longrightarrow \bar{W}_\chi^{\mathfrak{s}},$$

where $\bar{W}_\chi^{\mathfrak{s}}$ is "the finite W-algebra" of $\mathfrak{s}$:

$\mathfrak{s}$ is the Levi subalgebra of a parabolic subalgebra $\mathfrak{p}$, such that $\mathfrak{n}^- \subset \mathfrak{m} \subset \mathfrak{p}^-$, where $\mathfrak{n}^-$ is the nilradical of the opposite parabolic $\mathfrak{p}^-$.

$\bar{W}_\chi^{\mathfrak{s}} = (U(\mathfrak{s}) \otimes_{U(\mathfrak{m}^{\mathfrak{s}})} C_\chi)^{\mathfrak{m}^{\mathfrak{s}}}$, where $\mathfrak{m}^{\mathfrak{s}} = \mathfrak{m} \cap \mathfrak{s}$, χ is the restriction of χ on $\mathfrak{s}$.

3) One can show that if χ is regular, then $U(\mathfrak{g}_0)$ (correspondingly, $\bar{W}_\chi^{\mathfrak{s}}$) satisfies Amitsur–Levitzki identity. Hence W_χ satisfies Amitsur–Levitzki identity.

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