

# Momentum entanglement in relativistic quantum mechanics

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## Abstract

A new group-theoretic access to the interaction mechanism in elementary particle physics is presented.

Within an irreducible unitary two-particle representation of the Poincaré group, the commutation relations of the Poincaré group require that the two-particle states are momentum entangled.

As in gauge theories, momentum entanglement defines a correlation between two particles that can be described as an interaction provided by the exchange of virtual (gauge) quanta.

The coupling constant of this interaction is uniquely determined by the structure of the irreducible two-particle state space. For two massive spin-half particles the coupling constant matches the empirical value of the electromagnetic coupling constant.

# Introduction

## Noether's theorem

Symmetry  $\rightarrow$  conservation law  
(momentum and angular momentum)

Complementary dynamical law

Symmetry  $\rightarrow$  non-conservation law  
(exchange of momentum)

Interaction

Conservation laws + Dynamical law  $\rightarrow$  Interaction

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## Two particles

### Uncorrelated particles

Two particles in two **independent** space-time domains:

### Correlated particles

Two particles in the same space-time domain:

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Eigenstate of total momentum  $p$ :

particle momenta  $p_1, p_2$  **may** be entangled, with  $p_1 + p_2 = p$



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### Orbital angular momentum

Defined by relative momentum and distance:

$$m^{\mu\nu} = x^\mu(p_2^\nu - p_1^\nu) - x^\nu(p_2^\mu - p_1^\mu), \quad x^\mu = x_2^\mu - x_1^\mu$$

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
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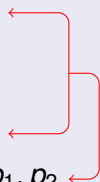
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Eigenstate of total and angular momentum:

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Particle momenta  $p_1, p_2$  **must** be entangled.



## Two correlated particles

### Eigenstate of total and angular momentum ...

... is a momentum entangled superposition of product states

$$|\mathbf{p}, m\rangle = \int_{\Omega} d\mathbf{k} c_{pmk} |\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k}\rangle, \quad \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}$$

### Irreducible representation

Eigenstates of total and angular momentum form a basis.  
General rule: Two-particle states are momentum entangled.

### Dynamical law

In irreducible two-particle representations of the Poincaré group the particles exchange *virtual quanta*.

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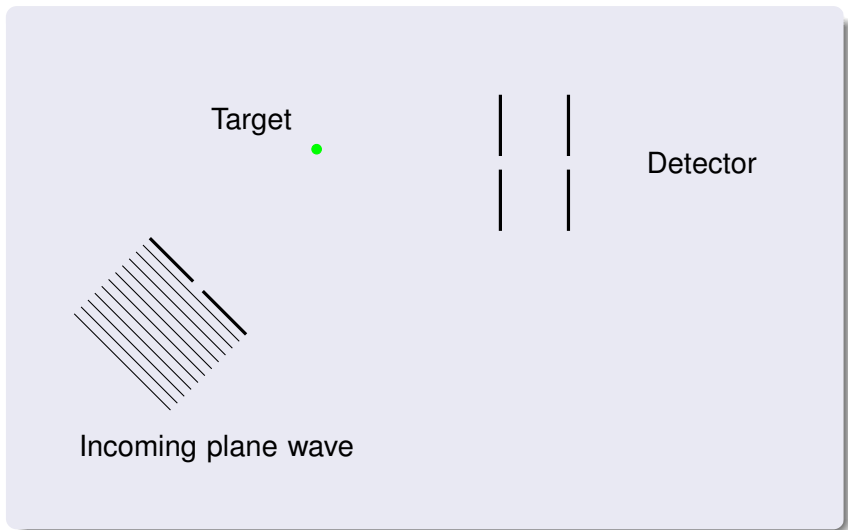
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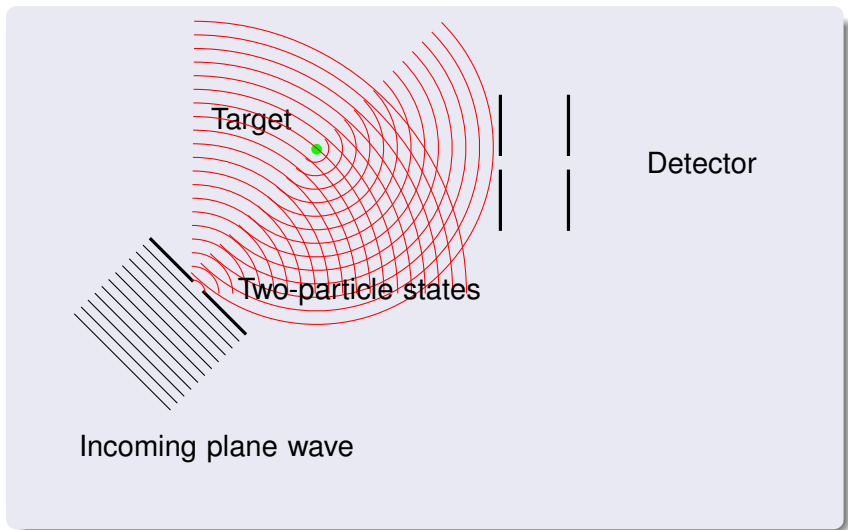
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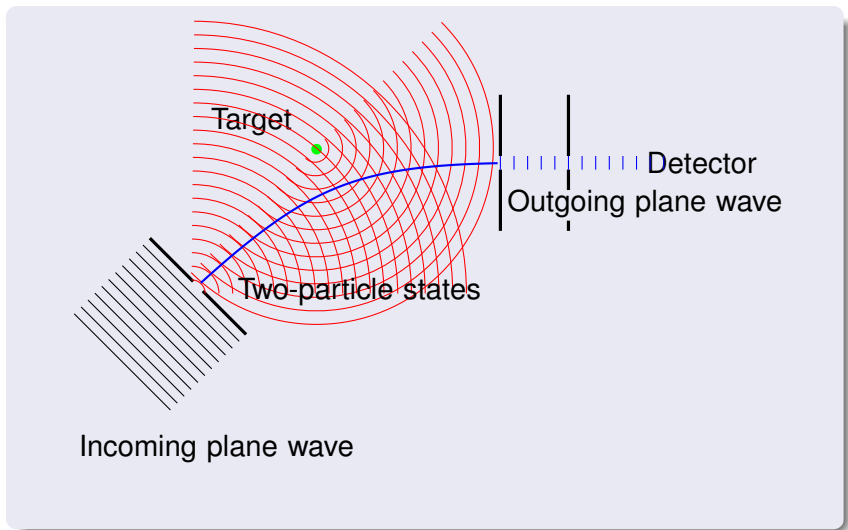
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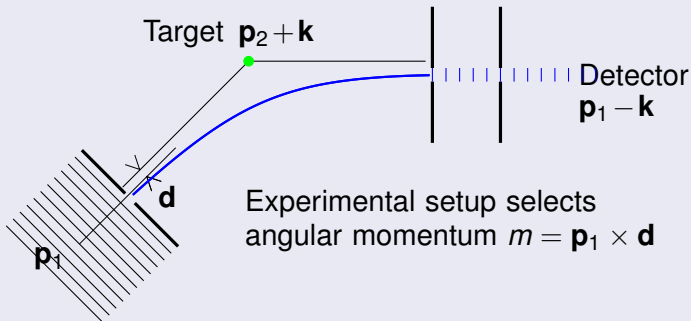
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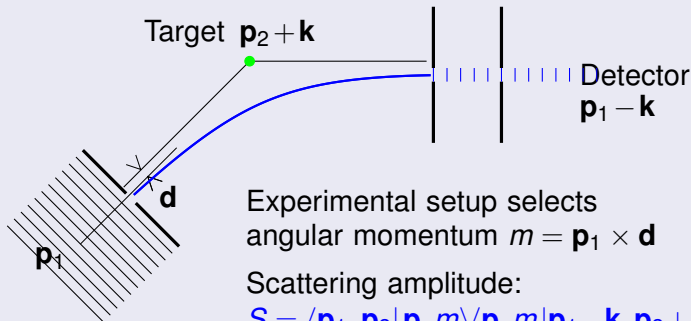


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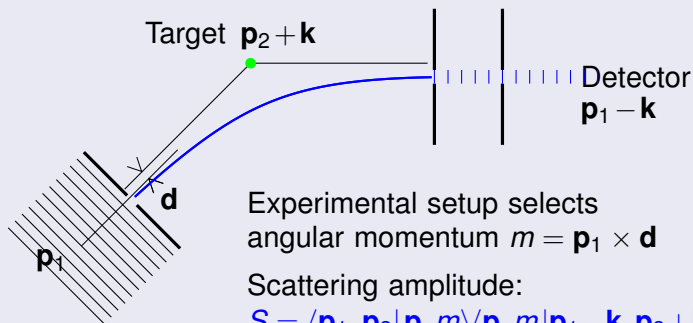




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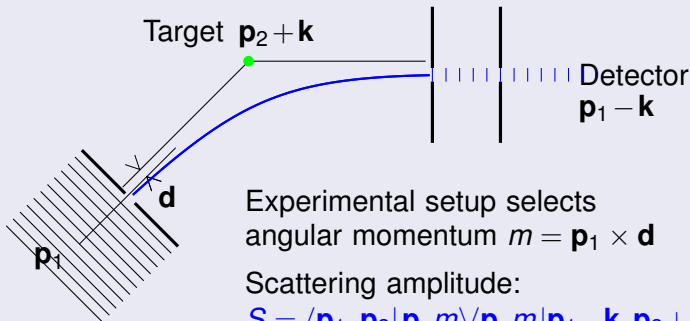
Experimental setup selects angular momentum  $m = \mathbf{p}_1 \times \mathbf{d}$

Scattering amplitude:

$$S = \langle \mathbf{p}_1, \mathbf{p}_2 | \mathbf{p}, m \rangle \langle \mathbf{p}, m | \mathbf{p}_1 - \mathbf{k}, \mathbf{p}_2 + \mathbf{k} \rangle$$

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$\implies$  Interaction by exchange of momentum

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## Parameter space $\Omega$

Derives from two-particle mass hyperboloid  $(p_1 + p_2)^2 = M^2$

## Two-particle state

$$|\Phi\rangle = \int_{\Omega} d\omega(\mathbf{p}_1, \mathbf{p}_2) c_{\mathbf{p}_1 \mathbf{p}_2} |\mathbf{p}_1, \mathbf{p}_2\rangle$$

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$$\implies \omega^2 = \text{coupling constant}$$

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$$\omega^2 = 8\pi V(D^5)^{\frac{1}{4}} / (V(S^4) V(Q^5)) \text{ Armand Wyler (1971) [wyl]}$$

$\omega^2 = \alpha$  identifies interaction as electromagnetic interaction

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## Relation to gauge theories

### Standard Model: Electromagnetic interaction ...

... results from gauge invariance,  
which requires the coupling to a gauge field,  
causing the exchange of virtual gauge bosons,  
causing momentum entangled two-particle states.

### Dynamical Law: Electromagnetic interaction ...

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[www.researchgate.net/publication/264117671](http://www.researchgate.net/publication/264117671)

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Thank you for your attention

# Quantum electrodynamics

## Basic 2-particle interaction ...

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### Problems of perturbation algorithm

Implements the entangled structure of angular momentum eigenstates – additively – on top of the product representation, rather than – subtractively – by projection onto irrep.

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### Same interaction mechanism for spinless particles

$$[p^\nu, m^{\mu\nu}] \neq 0 \neq [p^\mu, m^{\mu\nu}]$$

No dependency on  $\hbar \Rightarrow$  Mechanism preserved in classical limit.  
 Exchange of momentum = Momentum flow = Off-diagonal elements (traceless part) of energy-momentum tensor  
 $\Rightarrow$  Equivalent to curvature, described by the traceless part of the Riemann tensor (= Weyl tensor)  $\Rightarrow$  Conformal Gravity

### Effective coupling constant

No neutralization of "charges":

- $\Rightarrow$  A given particle couples potentially to  $10^{80}$  particles.
- Contribution of a specific second particle to S matrix:  $10^{-40}$
- $\Rightarrow$  Strength of gravitational interaction: Very weak

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